Inversion of Electrical Conductivity Parameters in Double-Layered Earth with 3-Dimensional Anomalies

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- Dec. 7   Questions
- Dec. 8   Brief presentation 1
- Dec. 9   Brief presentation 2
- Dec. 11 **Solved**!
- Future works: Inverse problem

- Maxwell’s equations in the frequency domain can be written as follows:

$$\nabla \times \vec{E} = i\omega\omega_0 \vec{H}$$

$$\nabla \times \vec{H} = \vec{J} + (\sigma - i\omega\varepsilon_0)\vec{E}$$

- According to dyadic Green’s function theory, the solution of electric field strength is equivalent to that of the following integral equation:

$$E(R) = E_p(R) + i\omega\mu_0 \int_{V_A} (\sigma - \sigma_b)\overline{\overline{G}}_e(R, R') \cdot \vec{E}(R')dV'$$

- To facilitate numerical calculation, the study area is divided into unit cubes, and the conductivity of the discrete element is set to a constant, that is, each unit cube has a constant electric field. The integral equation can then be rewritten as the following form of discrete units:

$$E(R) = E_p(R) + i\omega\mu_0 \sum_{n=1}^{N} (\sigma_n - \sigma_b) \int_{V_n} \overline{\overline{G}}_e(R, R') dV' \cdot \vec{E}_n$$

- The focal point of the electric field for the m-th unit cube can be expressed as:

$$E_m = E_{pm} + i\omega\mu_0 \sum_{n=1}^{N} (\sigma_n - \sigma_b) \int_{V_n} \overline{\overline{G}}_e(R_m, R') dV' \cdot \vec{E}_n$$

- Γ_{mn} is the dyadic Green's function for a cubic volume of current. We write Γ_{mn} as the sum of two components.

$$\Gamma_{mn} = \int_{V_n} \overline{\overline{G}}_e(R_m, R')dV'$$

$$\Gamma_{mn} = \Gamma_{mn}^P + \Gamma_{mn}^S$$

- Γ_{mn}^P can be shown that,

$$\Gamma_{mn}^P = \int_{V_n} (\overline{\overline{I}} + \frac{1}{k^2}\nabla\nabla)\frac{e^{ikR}}{4\pi R}dV'$$

- then

$$\Gamma_{mn}^P = \Gamma_{amn}^P + \Gamma_{bmn}^P$$

- with

$$\Gamma_{amn}^{P} = \int_{V_n} \bar{\bar{I}} \frac{e^{ikR}}{4\pi R} dV'$$

$$\Gamma_{bmn}^{P} = \frac{1}{k^2} \int_{V_n} \nabla\nabla \frac{e^{ikR}}{4\pi R} dV'$$

- Approximate analytic expression of Γ_{amn}^{P} :

$$\Gamma_{amn}^{P} = \bar{\bar{I}} \begin{cases} \frac{1}{k^2}[(1-ika)e^{ika} - 1] & m = n \\ \frac{e^{ikR_{mn}}}{k^2 R_{mn}}(\sin ka - ka\cos ka) & m \neq n \end{cases}$$

- Then a typical diagonal element of Γ_{bmn}^{P} is given by

$$\Gamma_{bmn}^{Pd} = \frac{1}{4\pi k^2}\sum_{l=1}^{2}(-1)^l[Q_l + (\frac{k^2\Delta^2}{2R} + i\frac{k^3\Delta^2}{3})(x_m - x_n - (-1)^l\frac{\Delta}{2})]$$

- A typical off-diagonal element of Γ_{bmn}^{P} is given by

$$\Gamma_{bmn}^{Pod} = \frac{1}{4\pi k^2}\sum_{l=1}^{2}(-1)^l[T_l + (\frac{k^2\Delta^2}{2R} + i\frac{k^3\Delta^2}{3})(x_m - x_n)]$$

- Each component of Γ_{mn}^{S} can be shown that,

$$\Gamma_{mn}^{S(xx)} = \frac{i\Delta^3}{4\pi}\left\{\sin\phi_{mn}\cdot\gamma_1 - \cos\phi_{mn}\cdot\gamma_4 + \frac{\cos^2\phi_{mn} - \sin^2\phi_{mn}}{\rho_{mn}}(\gamma_2+\gamma_5)\right\}$$

$$\Gamma_{mn}^{S(yx)} = \frac{i\Delta^3}{4\pi}\sin\phi_{mn}\cos\phi_{mn}\cdot\left\{-(\gamma_1+\gamma_4) + \frac{2}{\rho_{mn}}(\gamma_2+\gamma_5)\right\}$$

$$\Gamma_{mn}^{S(zx)} = \frac{i\Delta^3}{4\pi}\cdot\frac{i\cos\phi_{mn}}{k^2}\cdot\gamma_6$$

$$\Gamma_{mn}^{S(xy)} = \Gamma_{mn}^{S(yz)}$$

$$\Gamma_{mn}^{S(yy)} = \frac{i\Delta^3}{4\pi}\left\{\cos^2\phi_{mn}\cdot\gamma_1 - \sin^2\phi_{mn}\cdot\gamma_4 - \frac{\cos^2\phi_{mn} - \sin^2\phi_{mn}}{\rho_{mn}}(\gamma_2+\gamma_5)\right\}$$

$$\Gamma_{mn}^{S(zy)} = \frac{i\Delta^3}{4\pi}\cdot\frac{i\sin\phi_{mn}}{k^2}\cdot\gamma_6$$

$$\Gamma_{mn}^{S(xz)} = -\Gamma_{mn}^{S(zx)}$$

$$\Gamma_{mn}^{S(yz)} = -\Gamma_{mn}^{S(zy)}$$

$$\Gamma_{mn}^{S(zz)} = \frac{i\Delta^3}{4\pi}\cdot\frac{1}{k^2}\cdot\gamma_3$$

- with

$$\gamma_1 = \int_0^\infty \frac{1}{h} a e^{ih(z_m+z_n)} \cdot \lambda J_0(\lambda\rho_{mn}) d\lambda$$

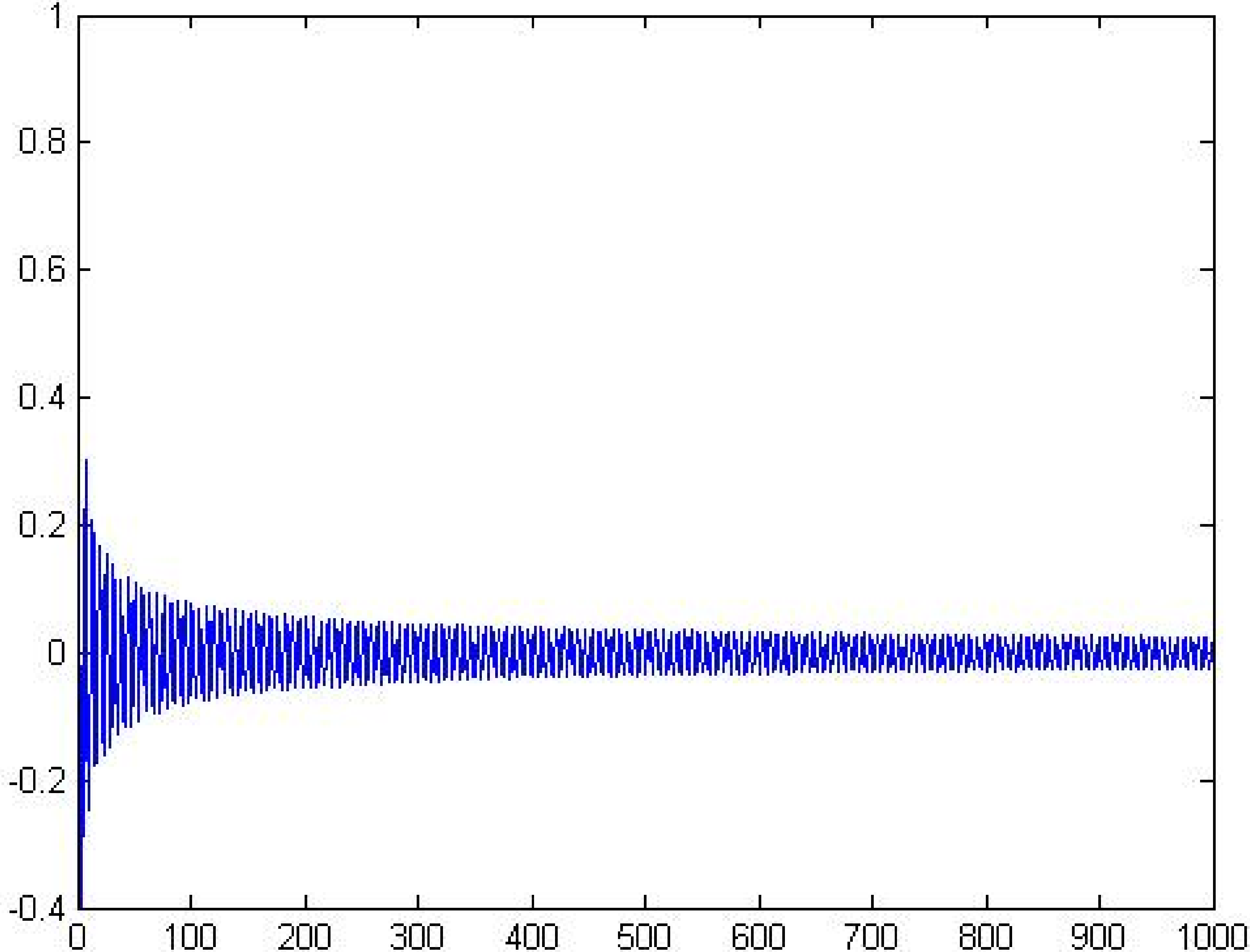
$$\gamma_2 = \int_0^\infty \frac{1}{\lambda h} a e^{ih(z_m+z_n)} \cdot \lambda J_1(\lambda\rho_{mn}) d\lambda$$

$$\gamma_3 = \int_0^\infty \frac{1}{h} \lambda^2 b e^{ih(z_m+z_n)} \cdot \lambda J_0(\lambda\rho_{mn}) d\lambda$$

$$\gamma_4 = \int_0^\infty \frac{1}{h} b(1-\frac{\lambda^2}{k^2}) e^{ih(z_m+z_n)} \cdot \lambda J_0(\lambda\rho_{mn}) d\lambda$$

$$\gamma_5 = \int_0^\infty \frac{1}{\lambda h} b(1-\frac{\lambda^2}{k^2}) e^{ih(z_m+z_n)} \cdot \lambda J_1(\lambda\rho_{mn}) d\lambda$$

$$\gamma_6 = \int_0^\infty \lambda b e^{ih(z_m+z_n)} \cdot \lambda J_1(\lambda\rho_{mn}) d\lambda$$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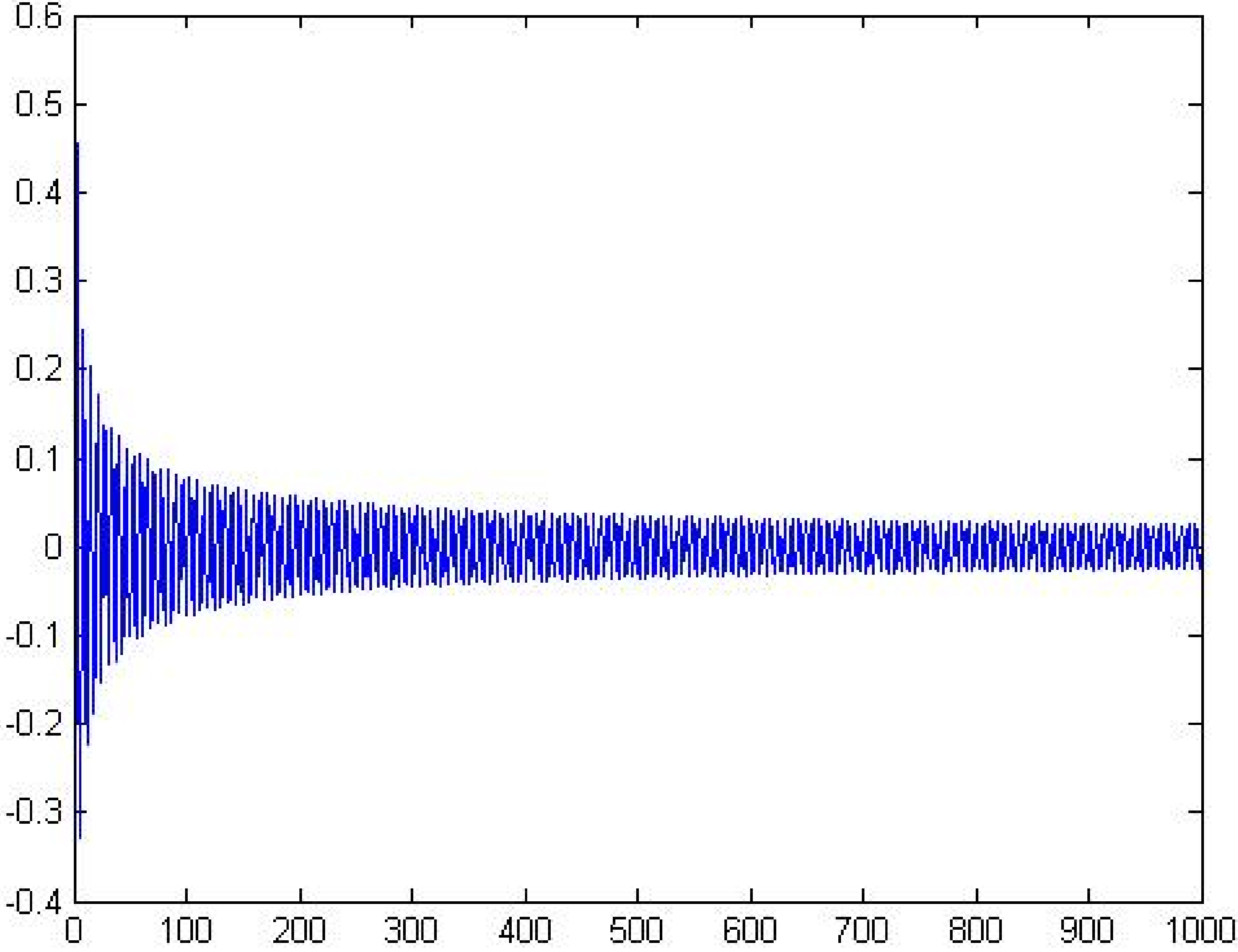
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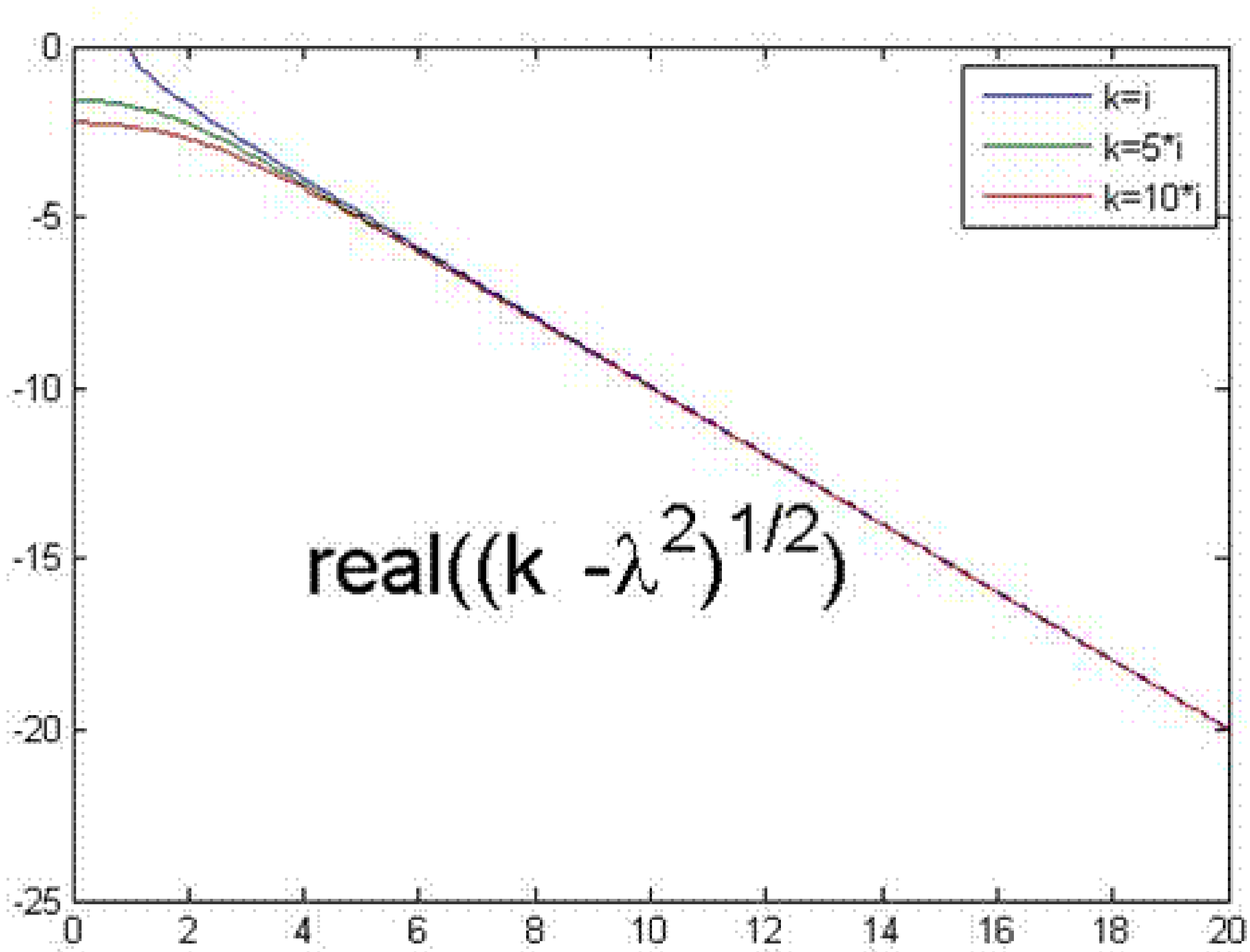

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-10
-15
-20
-25
0
2
4
6
8
10
12
14
16
18
20
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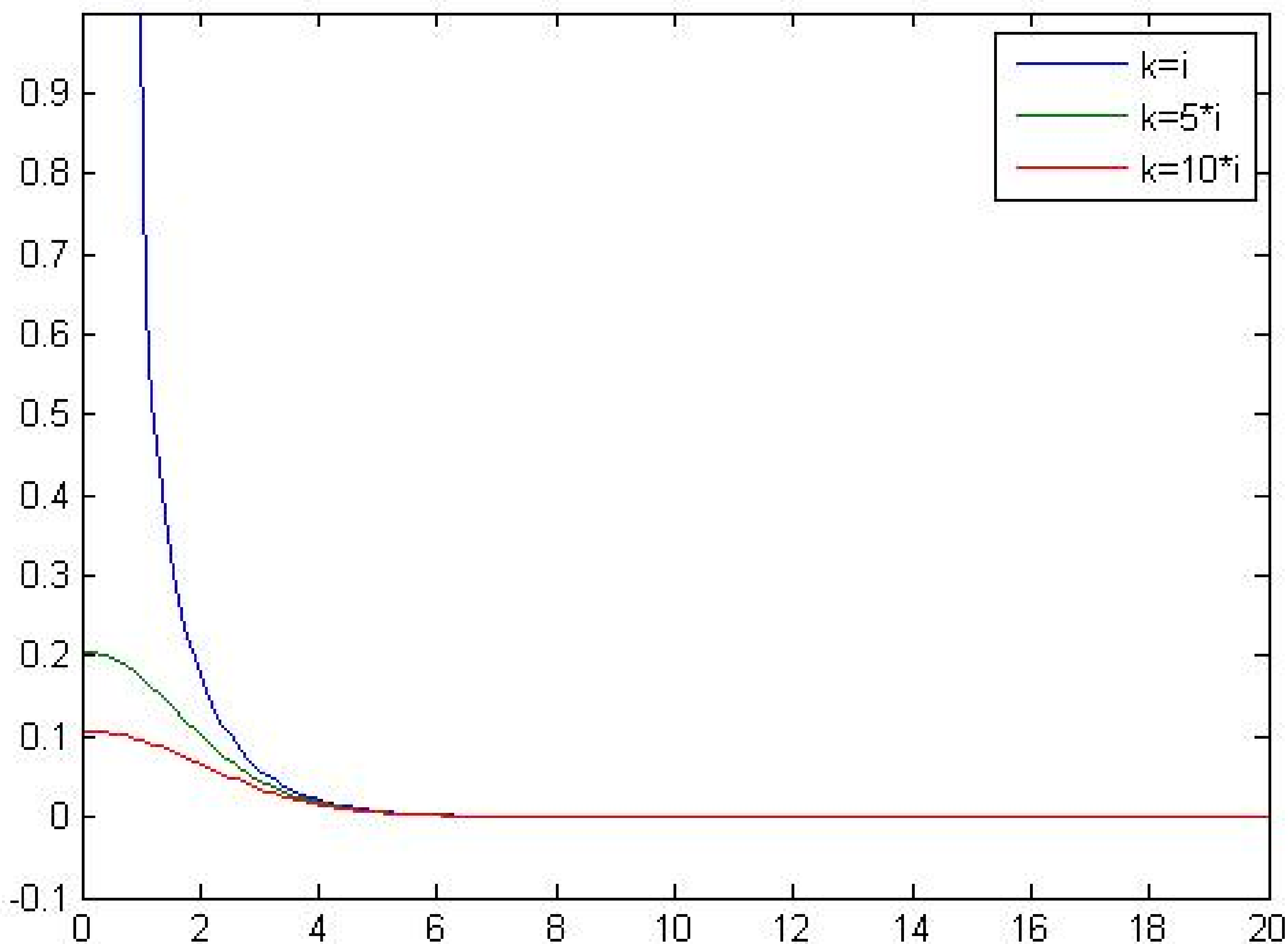
k=5*i
k=10*i
real((k -λ^2)^1/2)



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k=5*i
k=10*i
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0.7
0.6
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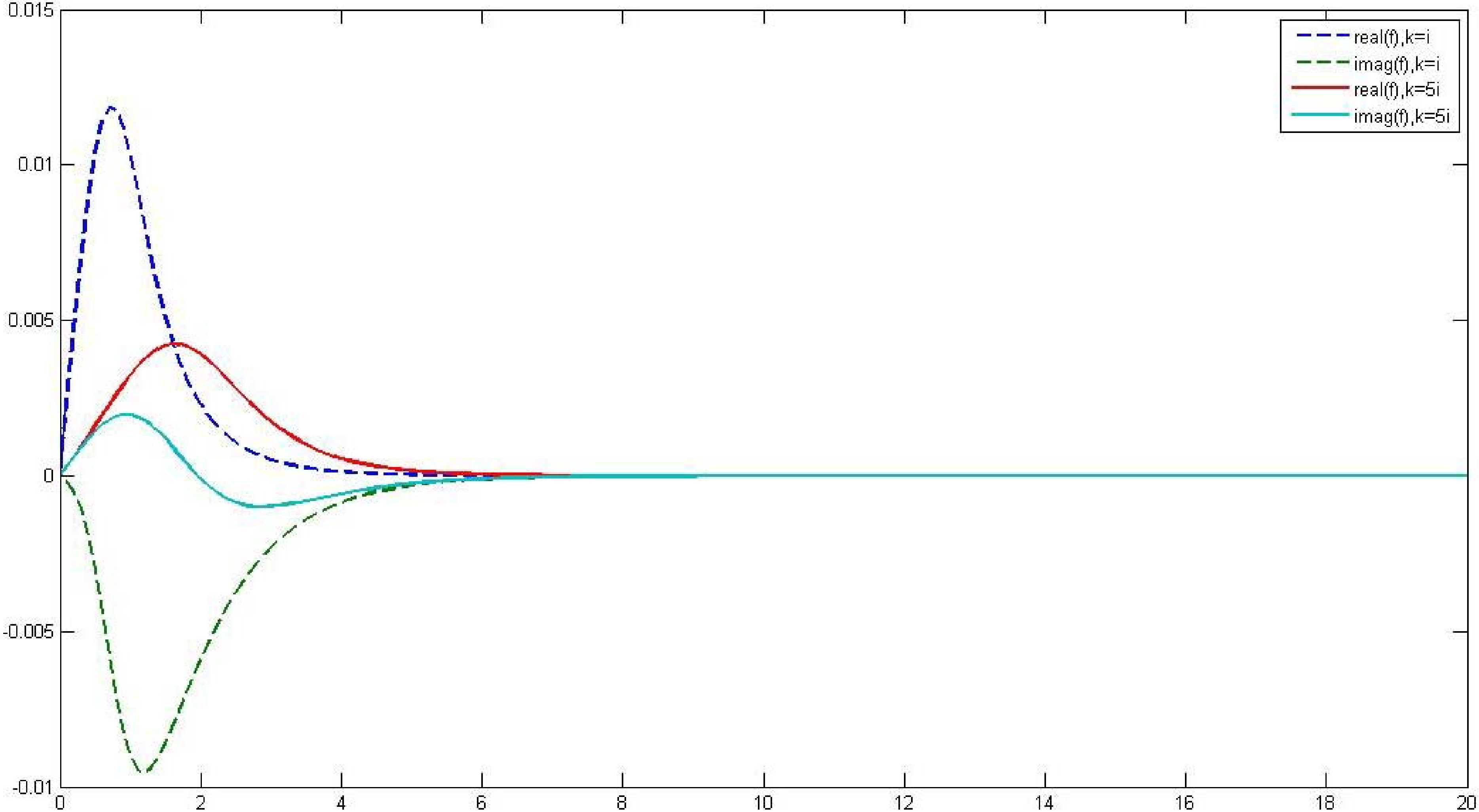
0
2
4
6
8
10
12
14
16
18
20



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0.01
0.005
0
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-0.01
0
2
4
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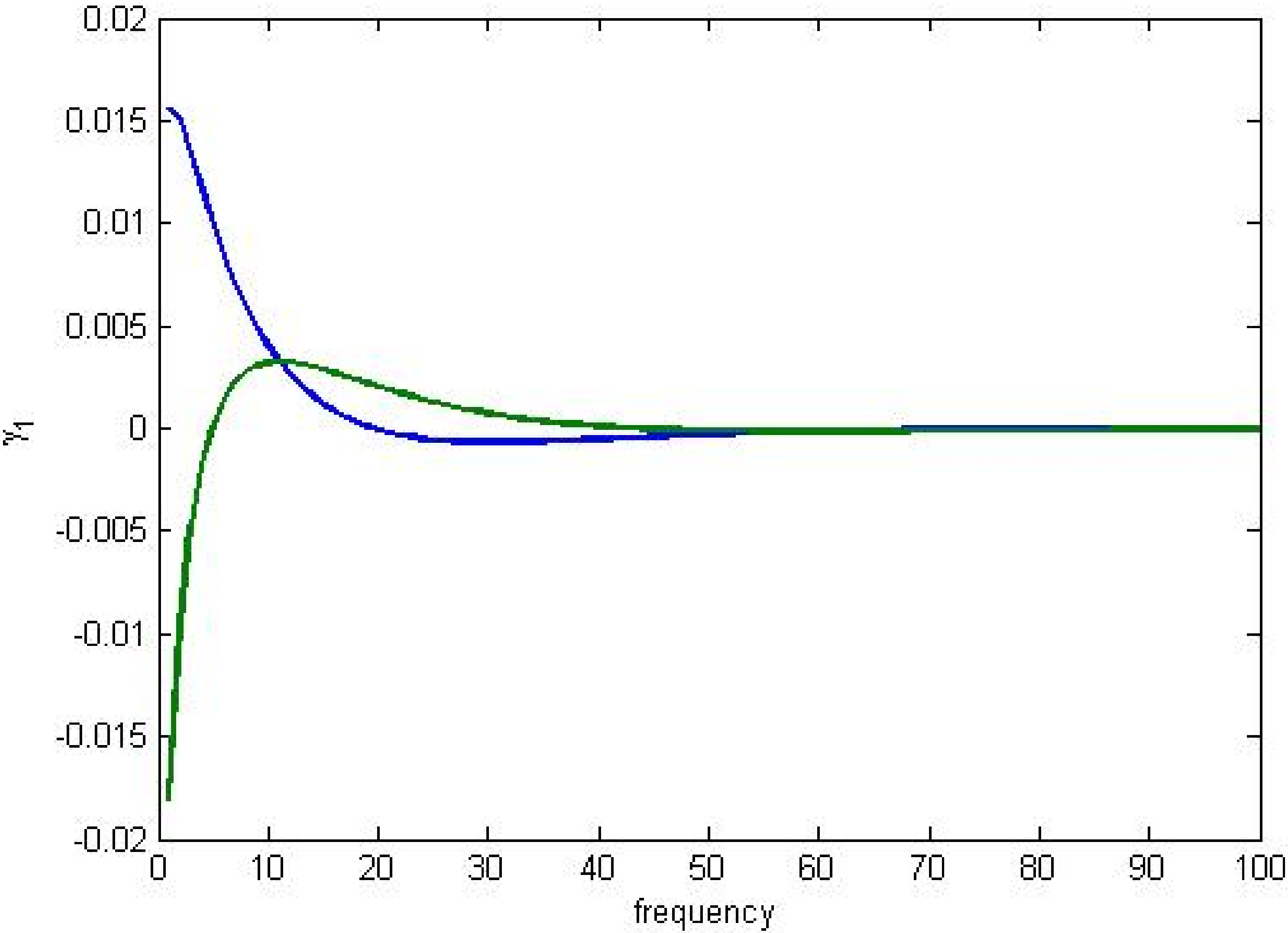
16
18
20
real(f),k=i
imag(f),k=i
real(f),k=5i
imag(f),k=5i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γ_1
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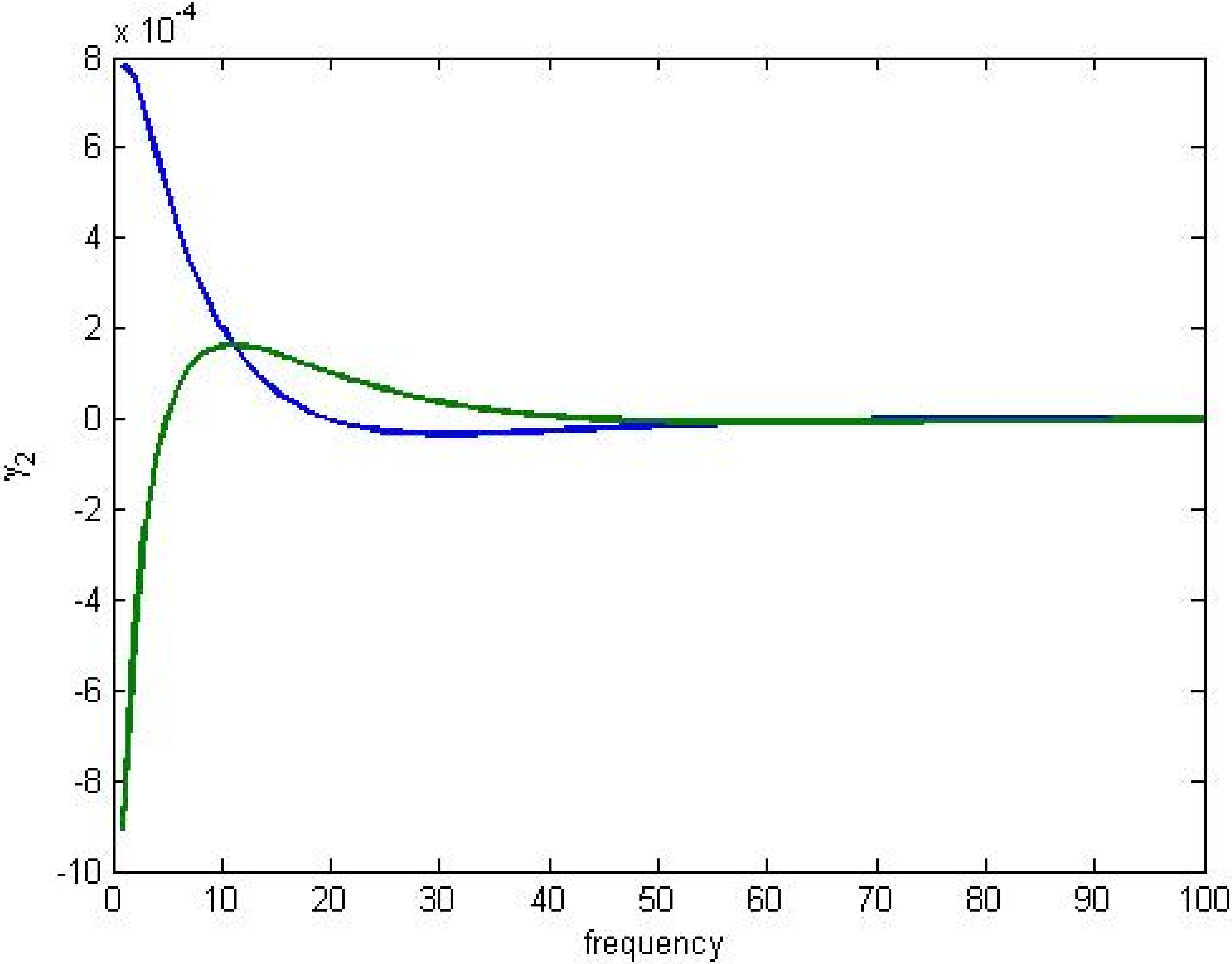
70
80
90
100
frequency



x 10^-4
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6
4
2
0
-2
-4
-6
-8
-10
γ2
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10
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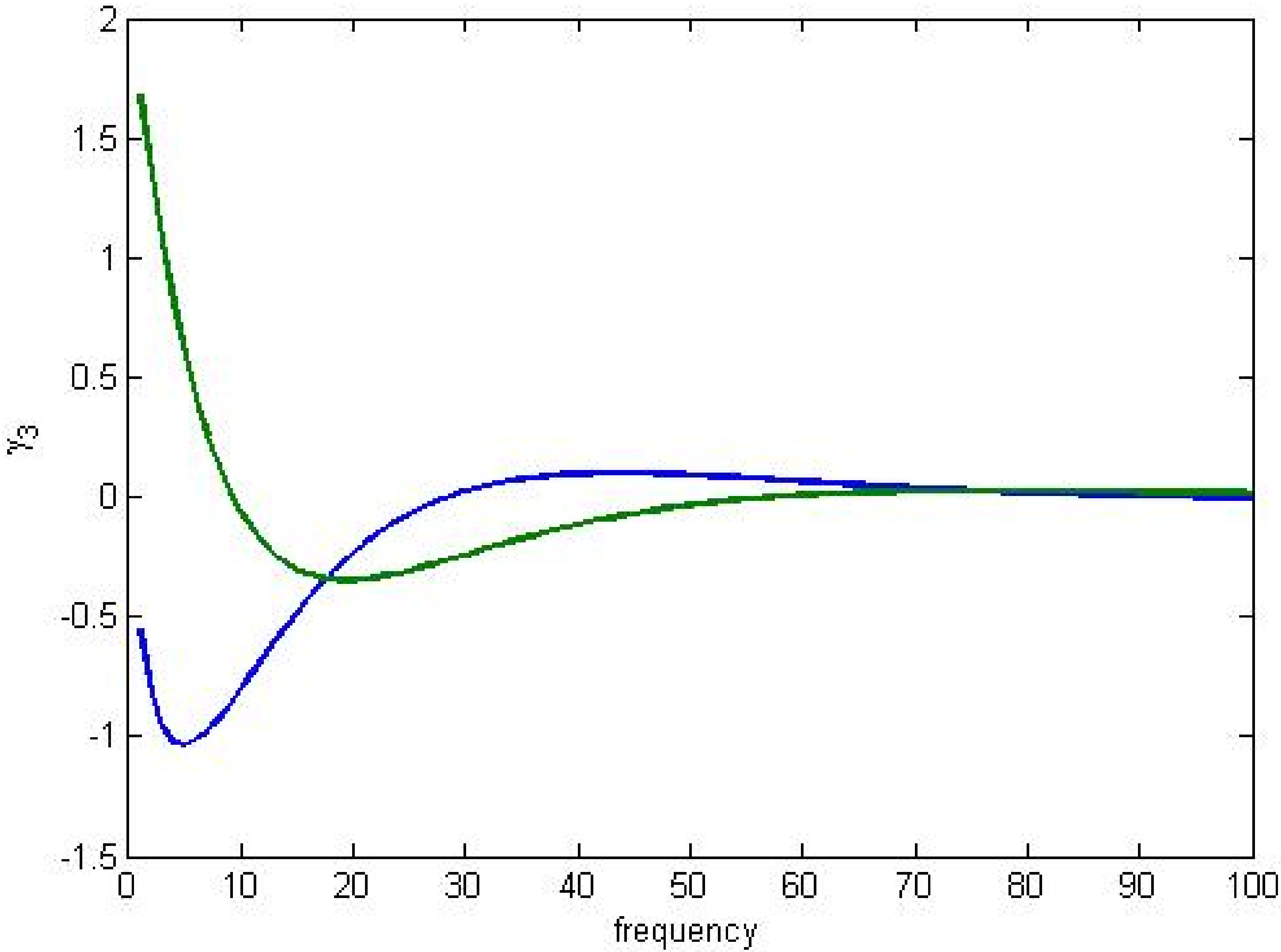
40
50
60
70
80
90
100
frequency



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1
0.5
0
-0.5
-1
-1.5
γ3
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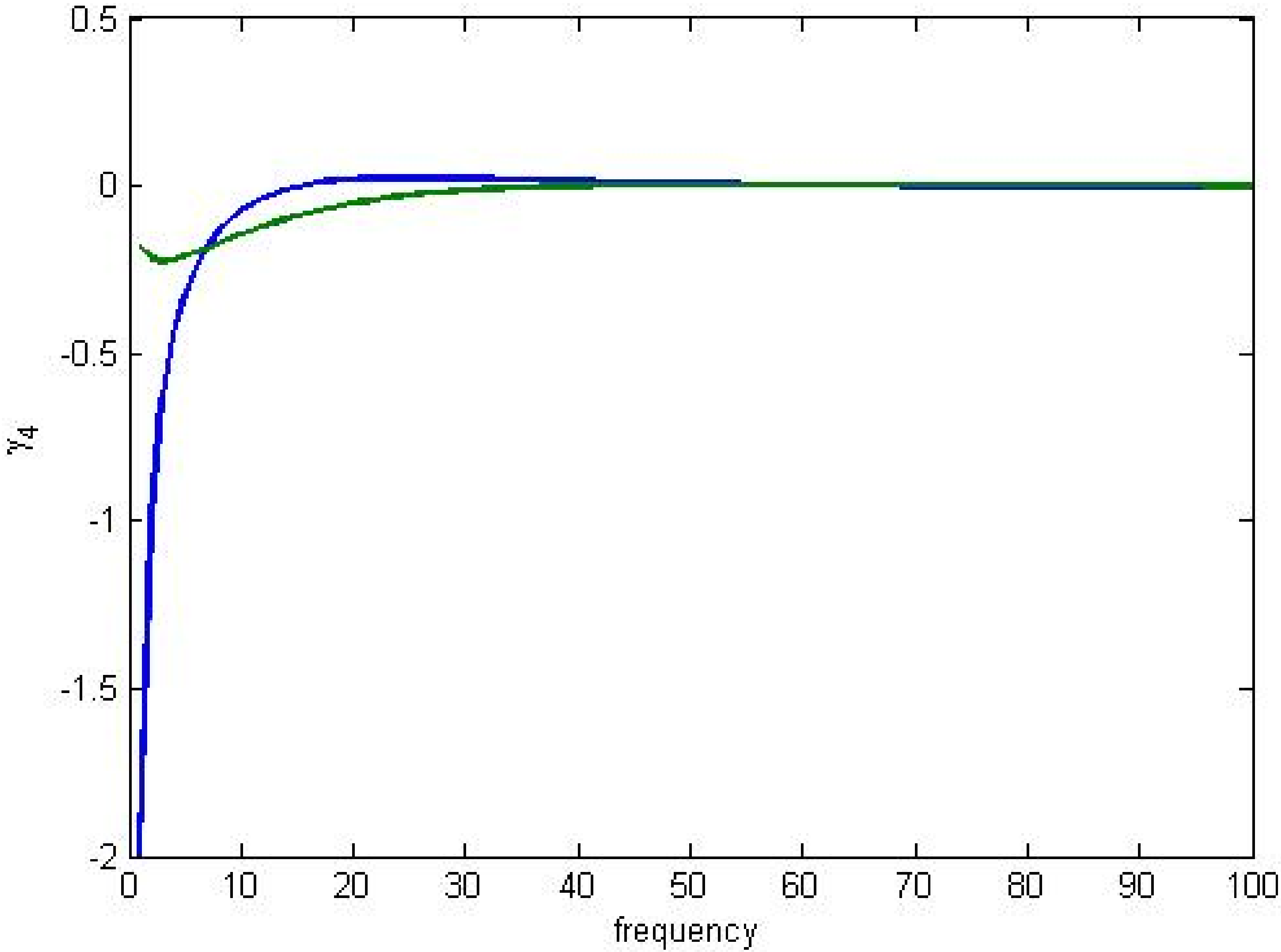
10
20
30
40
50
60
70
80
90
100
frequency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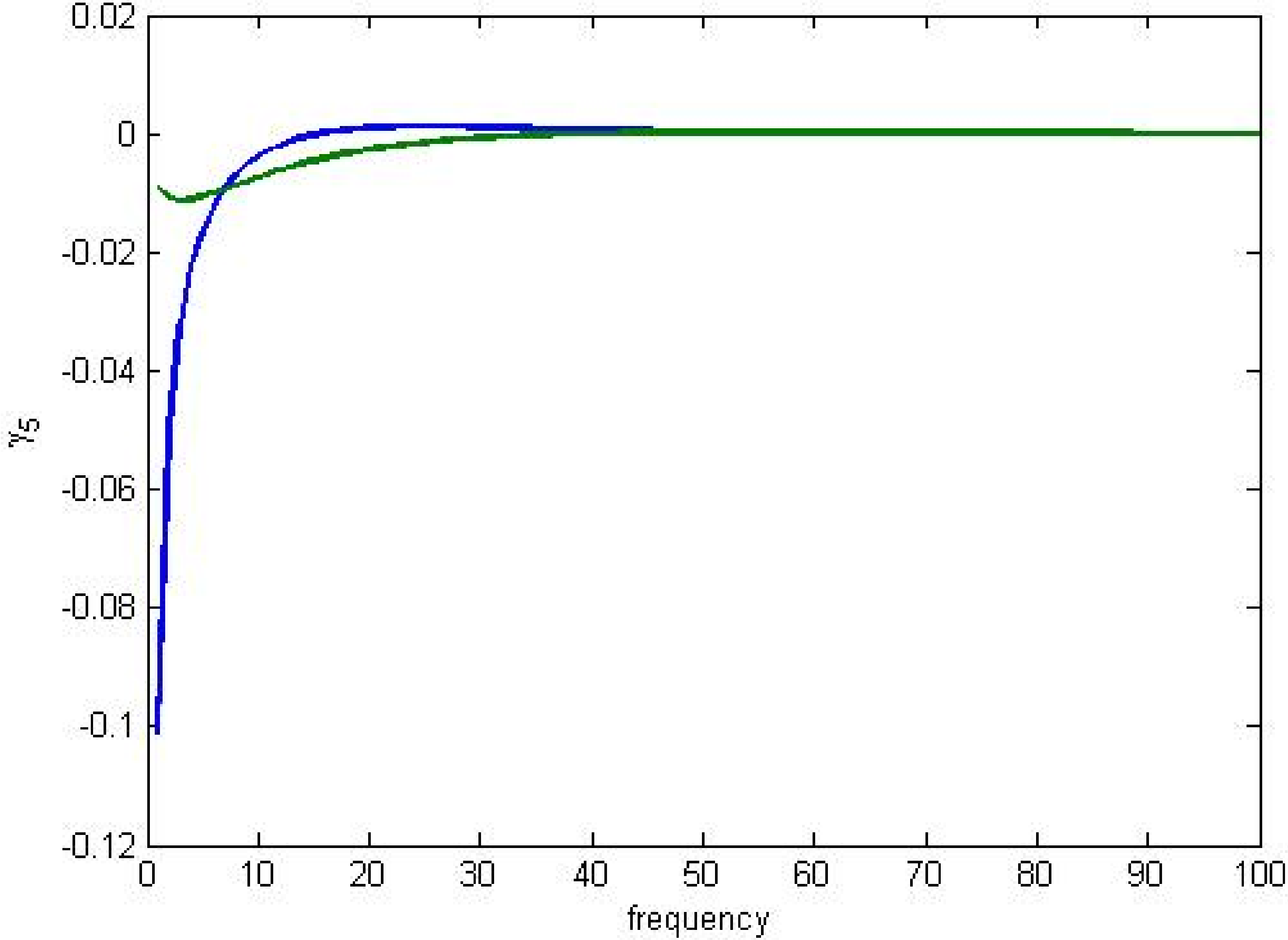
90
100
frequency
γ_4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γ_5
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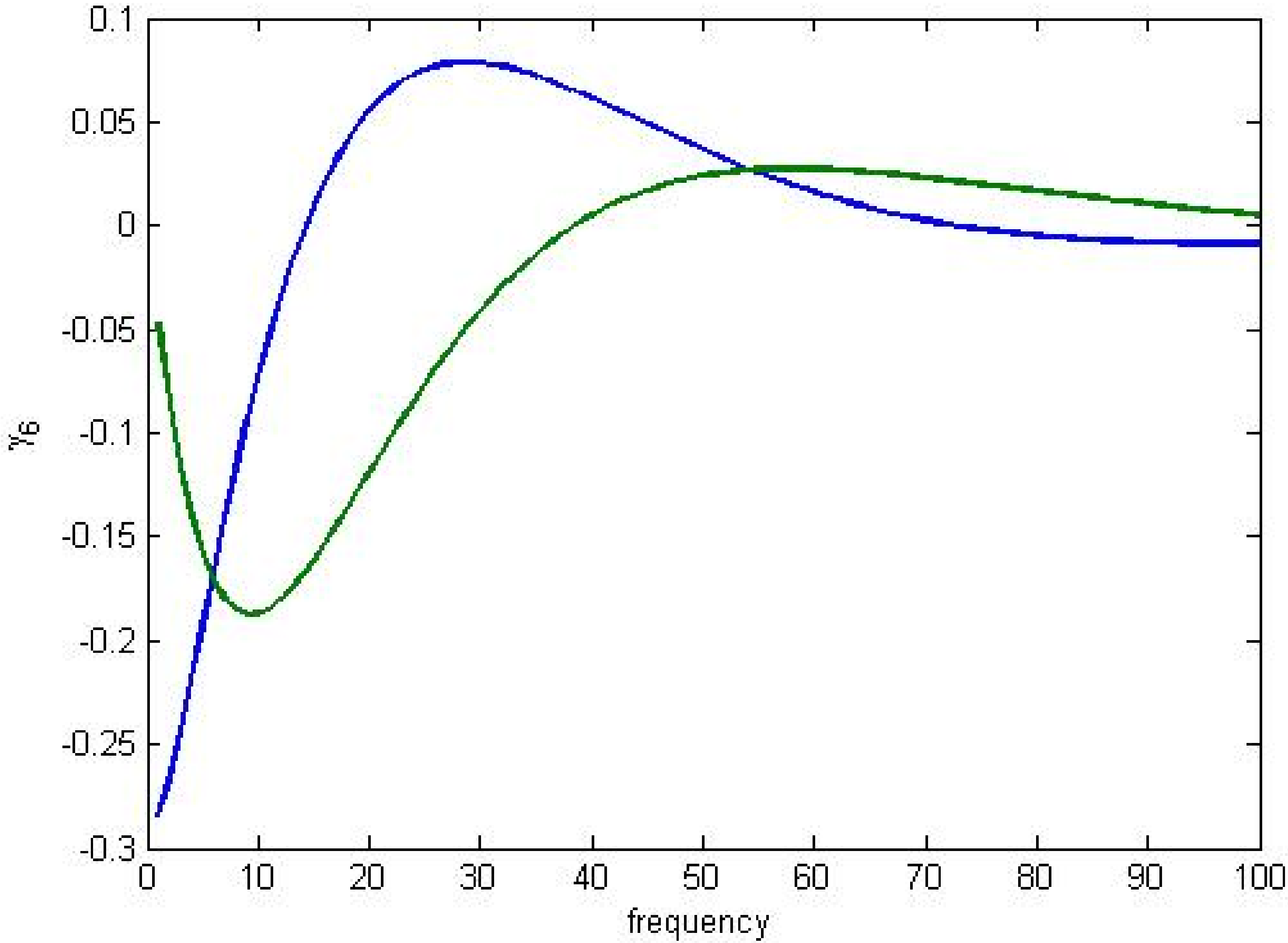
70
80
90
100
frequency



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0
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-0.1
-0.15
-0.2
-0.25
-0.3
γ6
0
10
20
30
40
50
60
70
80
90
100
frequency

Thanks for your kind attention!